

## CHAPTER 30

# Eye movements both reveal and influence problem solving

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### 6 Abstract

7 Within the context of the theory of embodied cognition, our most frequent motor movements—eye  
8 movements—are sure to play an important role in our cognitive processes. Not only do eye move-  
9 ments provide the experimenter with a special window into these cognitive processes, they provide  
10 the individual with a way to modify their cognitive processes. This chapter examines this dual role of  
11 eye movements (both revealing and modifying cognition) in a variety of problem solving tasks.  
12 Experimenters can better understand the underlying cognitive processes that are involved in prob-  
13 lem solving by recording eye movement patterns, and individuals can better perform their problem  
14 solving when they produce the right eye movement patterns.

### 15 Introduction

16 A well known puzzle, known as the river-crossing problem, places missionaries in boats with cannibals  
17 and asks you to transit them across a river in a manner that ensures that no missionary is eaten. Consider  
18 one such dire scenario: ‘Three missionaries and three cannibals want to get to the other side of a river.  
19 There is a small boat, which can fit only two. To prevent a tragedy, there can never be more cannibals  
20 than missionaries together.’ If the reader is like this chapter’s authors, this puzzle induces a flurry of  
21 dynamic imagery, including a visualization of the nervous missionaries, salivating cannibals, a boat, a  
22 river, and perhaps even irrelevant factors like the protagonists’ appearance, or a sun in the sky. The  
23 reader, like us, may manipulate the visualization and begin moving its ‘pieces’ in seeking a solution.

24 General solutions to versions of this problem have been proposed since the 19th century (Pressman  
25 and Singmaster, 1989), and into the mid-20th century in a burgeoning field of artificial intelligence  
26 (Amarel, 1968). These solutions have never attempted to integrate the rich ‘perceptual simulation’  
27 (Barsalou, 1999) noted in the previous paragraph. In general, they have had highly formal logical  
28 characteristics, translating relevant parts of these problems into variables and operators often dubbed  
29 ‘amodal’ because they lack the characteristics of sensorimotor modalities involved in imagery. This  
30 has been a general feature of ‘classical’ approaches to our cognitive system: treating our internal  
31 thought processes as governed by abstract symbols not much different from a digital computer’s  
32 states and processes (e.g. Newell, 1990; Newell and Simon, 1961). To the uninitiated, this may seem  
33 like a very complex, mathematical way of approaching these issues as potential explanations for how

1 we ourselves solve them. But in fact this strategy has been an influential simplification, allowing  
 2 cognitive scientists to develop computer programs that perform human-like problem solving in a  
 3 diverse range of problem spaces (see Poole et al., 1997, for many examples).

4 There has been considerable debate about the usefulness of these formalistic approaches to under-  
 5 standing human cognition in general (e.g. Dreyfus, 1979; **Marcus, 2001**; Searle, 1980; Spivey, 2007).  
 6 One charge against the classical approach has been its lack of perceptual and motor detail that we seem  
 7 richly to experience in day-to-day cognitive functioning, as the first paragraph of this chapter attempts  
 8 to demonstrate (see, e.g. Clark, 1997, for review). In the past two decades, the view that cognitive  
 9 scientists should integrate sensorimotor, or ‘modal,’ representations in our theories of cognition has  
 10 been gaining traction. Often dubbed the ‘embodiment of cognition,’ the approach sees cognitive  
 11 processes as directly involving modal sensorimotor systems, e.g. visual, somatosensory, and auditory  
 12 perception, as well as oculomotor and skeletomotor planning. In other words, even during ‘high-level’  
 13 cognitive processes such as problem solving, the brain recruits ‘low-level’ sensorimotor neural subsys-  
 14 tems to assist in the cognitive computations.

AQ: please supply Marcus 2001 reference



15 In this chapter, we review research showing that eye movements support the embodiment of  
 16 problem solving. When facing many kinds of puzzles and problems, our cognitive system makes  
 17 use of implicit perceptuomotor activities to search the problem space for solutions. In what follows,  
 18 we present a selective review of evidence for embodiment especially relevant to eye movements  
 19 and problem solving. This review motivates two key predictions: Eye movements should: 1) reveal  
 20 problem solving processes as they unfold, and 2) potentially aid problem solving directly. We then  
 21 showcase recent empirical support for both predictions. The chapter ends with theoretical discus-  
 22 sion, and future directions in which eye movement and other dynamic, temporal methodologies  
 23 could contribute to our understanding of high-level cognition.

## 24 **The embodiment of cognition**

25 Consider again the river-crossing problem. This type of problem has some basic properties that  
 26 many puzzles and problems have in common. First, it is conveyed through linguistic means. Problem  
 27 solvers are provided statements that lay out a set of premises, assumptions, arrangements, etc. that  
 28 are intended to establish basic rules or constraints on a scenario. Second, a given scenario may have  
 29 some spatiotemporal conditions that approximate arrangements of things in the natural world. The  
 30 river-crossing problem has folks travelling in boats, while other problem spaces involve moving blocks  
 31 about pegs (e.g. Tower of Hanoi problem: Simon, 1975), or connecting dots on a two-dimensional  
 32 surface (e.g. nine-dot problem: MacGregor et al., 2001). Both of these qualities—linguistic transmis-  
 33 sion and spatiotemporal arrangement—invoke cognitive processes that have embodied sensorimo-  
 34 tor qualities. Evidence for this has come directly from eye tracking experiments.

35 In an obvious way, concrete spatial language that explicitly mentions positions or directions of  
 36 movement may invoke sensorimotor representations. But even abstract figurative language can  
 37 generate embodied effects, evidenced in eye movement patterns. For example, Matlock and colleagues  
 38 have investigated the processing of sentences that figuratively invoke motion, known as ‘fictive  
 39 motion’ descriptions (Matlock, 2004). Matlock and Richardson (2004) conducted a passive viewing  
 40 experiment in which participants saw a scene of objects and listened to sentences. Some scene/  
 41 sentence pairs contained fictive motion. For example, a view of a desert with a road down the centre  
 42 of the scene could be described with ‘The road *runs through* the desert’ (fictive motion) or simply  
 43 ‘The road is in the desert’. Participants spent more time fixating the path of motion when they heard  
 44 fictive motion sentences than when they heard the literal control sentences. Richardson and Matlock  
 45 (2007) further demonstrated that when the sentences described movement difficulty (e.g. ‘... the  
 46 desert is rough’), participants’ eye movement patterns covaried with this movement difficulty.  
 47 Scanpaths exhibited enhanced focus on the region of the fictive motion (e.g. along the road) under  
 48 conditions of more difficult movement. These and numerous other studies have shown that a diverse  
 49 range of language processes make use of sensorimotor representations (for reviews, see Anderson  
 50 and Spivey, 2009; Barsalou, 2008; Glenberg and Kaschak, 2003).

1 As for spatiotemporal arrangement, river-crossing and other problems may induce visual imagery  
 2 of manipulating this arrangement, during which modal visual areas of cortex are activated (see  
 3 Kosslyn and Thompson, 2003; Kosslyn et al., 2003 for review). If this visual system activation is func-  
 4 tionally important to visual imagery, then oculomotor output is likely to correspond to the structure  
 5 of imagery. Indeed, Spivey and Geng (2001, expt. 1) have shown that visual imagery elicits eye move-  
 6 ments (while viewing a blank screen) that correspond to the locations and movement directions  
 7 in the spoken descriptions (Altmann, 2004; Brandt and Stark, 1997; Laeng and Teodorescu, 2002;  
 8 see also Altmann, Chapter 54, this volume).

9 There may be good reasons for this oculomotor enactment of cognition: memory systems make  
 10 use of external spatial locations as anchors for encoded information. The eyes should thus reflect  
 11 these external ‘indexes’ (Pylyshyn, 1989) for recalled information. Richardson and Spivey (2000)  
 12 have shown that when participants learn new facts in association with spatial regions on a screen,  
 13 participants look to those now-blank regions when recalling the facts. In a series of experiments,  
 14 participants saw a 2×2 grid, and in different cells viewed a video of a face presenting various state-  
 15 ments (e.g. ‘Australia’s capital is Canberra’). When participants were queried to verify a version of  
 16 one of those statements, they frequently looked to the corresponding empty cell in the grid while they  
 17 answered the query—even though they could easily tell in their peripheral vision that it was empty,  
 18 and the statement has been delivered auditorily anyway!

19 A natural question that follows is whether these eye movements to those locations can improve  
 20 memory for that object or event. That is, can recreating during recall the eye movement pattern that  
 21 took place during encoding instigate at kind of pattern completion process? It has recently been  
 22 hypothesized that if eye movements are systematically manipulated by the experimental task, e.g.  
 23 turned into an independent variable instead of a dependent variable, then one might observe different  
 24 memory performance for trials in which the eyes are fixating the correct location versus an incorrect  
 25 location (Ferreira et al., 2008). Although this is certainly a logical possibility, the existing data are not  
 26 consistent with it (Richardson et al., 2009). In every test for such an improvement in memory accuracy  
 27 conducted so far, none has been found (Hoover and Richardson, 2008; Richardson and Kirkham,  
 28 2004; Richardson and Spivey, 2000; Spivey and Geng, 2001, expt. 2). Recently, there is evidence for a  
 29 slight decrease in reaction time to memory probes when the eyes are fixating the correct location  
 30 compared to when they are fixating the wrong location, but no evidence for improvements in memory  
 31 accuracy (Theeuwes et al., in press; Vankov, 2009). For example, Vankov (2009) presented participants  
 32 with four line drawings of objects and then presented a memory probe consisting of a single word that  
 33 might refer to one of the objects. In some conditions, the word showed up in the same location as the  
 34 now-absent object, and in other conditions, it showed up in a different location. Vankov observed  
 35 some subtle reaction time differences between these conditions, but the accuracy of memory perform-  
 36 ance was not affected by the location of the memory cue. Thus, rather than these memory-induced eye  
 37 movements being evidence for an embodiment of cognition solely inside the sensorimotor system  
 38 (where pattern completion of neural patterns can take place), they may be better evidence for an  
 39 embeddedness of cognition in the environment (e.g. O’Regan and Noë, 2002; Spivey et al., 2004),  
 40 whereby a small bit of internal semantic information regarding an object or event is linked to a location  
 41 in the visual field where ‘the rest’ of the information is expected to be accessible (O’Regan, 1992).

42 The selective review above supports the involvement of embodied representations in language proc-  
 43 esses and visual imagery, both likely components of naturalistic problem solving. Memory processes  
 44 accompanying both of these can be anchored in spatial locations in the world, and eye-movement  
 45 signatures reveal this. In the next section, we briefly provide a more mechanistic account of the deep  
 46 connection between eye movements and cognition, which leads to two broad predictions about how  
 47 eye movements should be involved in problem solving.

## 48 **Eye movements are coextensive with cognition**

49 The key reason why eye movement patterns are so informative about mental activity is that they are  
 50 part and parcel of it. Rather than being a slave output system that patiently awaits discrete finalized

1 commands from the cognitive system, oculomotor processing is coextensive with cognitive process-  
 2 ing. The ongoing processes of cognition continuously ‘spill over’ (before a decision is final) into the  
 3 oculomotor system, causing it to prepare partially-active movement plans that are consistent with  
 4 the gradually accumulating perceptual evidence (Gold and Shadlen, 2007). This is due in part to the  
 5 fact that the eyes tend to make saccadic movements about every 200–300 ms, whereas it takes about  
 6 400–500 ms for a neural population code that embodies the representation of a visual object to  
 7 achieve its maximal activation (e.g. Rolls and Tovee, 1995). As a result, the eyes are often moving to  
 8 fixate a new object before the previous object has been fully and completely recognized. This contin-  
 9 uously flowing perception-action cycle (Neisser, 1976) becomes an autocatalytic causal loop, in  
 10 which cognition emerges (Spivey, 2007). Before a cognitive decision to respond to the stimulus envi-  
 11 ronment has reached completion, the eyes will often move to fixate a new object or location in the  
 12 visual field. This newly foveated perceptual information then changes the cognitive processes which  
 13 led to that eye movement—and which were on their way to one decision but now may be on their  
 14 way to a different one! Cognition somehow manages to simultaneously be a major cause of eye-  
 15 movement patterns and partly a result of them.

16 A concrete demonstration of this continuity between a cognitive process (such as a decision) and  
 17 an eye movement comes from a study by Gold and Shadlen (2000). As perceptual input accumulates  
 18 over time to produce an overt decision and motor response, the continuous evolution of that decision  
 19 can be seen in eye movement data. That is, the network of brain systems in which a perceptual deci-  
 20 sion is generated includes oculomotor nuclei, such as the frontal eye fields. Gold and Shadlen trained  
 21 a monkey to respond to a pseudorandom dot-motion stimulus by looking at an upper response target  
 22 when the central motion stimulus had upward motion in it, and at a lower response target when the  
 23 central motion stimulus had downward motion in it. Then, just before a voluntary eye movement  
 24 response was made, they microstimulated a region of the monkey’s frontal eye fields (via mild electric  
 25 current delivered through an electrode) to generate an involuntary rightward saccade. When the  
 26 monkey was allowed only a couple hundred milliseconds to see the motion stimulus, the involuntary  
 27 evoked saccade was a nearly pure rightward movement. However, with 300 or 400 or 500 ms to see  
 28 the motion stimulus, the involuntary evoked saccade showed more and more vertical component in  
 29 its direction. Essentially, the perceptual decision (of upward vs. downward motion) that was evolving  
 30 over several hundred milliseconds was simultaneously generating a pattern of neural activity in the  
 31 frontal eye fields that formed the beginnings of a voluntary upward or downward eye movement  
 32 command. When the microstimulation caused the involuntary rightward saccade, the partly evolved  
 33 voluntary eye movement command influenced the direction that the eyes actually went. Thus, rather  
 34 than the oculomotor system waiting until a single confident perceptual decision was achieved, it was  
 35 continuously ‘listening to’ and ‘involved in’ the gradual formation of that perceptual decision.

36 One might, in fact, wish to draw a distinction between the notions of the eye movement system  
 37 continuously *listening* to cognition versus being continuously *involved in* cognition. This kind of  
 38 distinction is often raised when discussing embodied cognition in general (Mahon and Caramazza,  
 39 2008), where some embodied cognition results may be best interpreted as a spreading of activation  
 40 from cognitive to motor subsystems (e.g. Glenberg and Kaschak, 2002), and other embodied cogni-  
 41 tion results clearly demonstrate a functional role for motor patterns feeding back into cognitive proc-  
 42 esses (Pulvermüller et al., 2005). Accordingly, the next two sections in this chapter will address  
 43 evidence for eye movements revealing certain cognitive processes related to problem solving tasks  
 44 because those cognitive processes continuously spread their activation patterns into the oculomotor  
 45 system, and also evidence for eye movements influencing the cognitive processes related to problem  
 46 solving tasks because those eye movement patterns change the way the cognitive processes function.

## 47 **Eye movements reveal problem solving**

### 48 **Mathematical problem solving**

49 A number of studies have shown a relationship between the activity of the eyes and the activity of the  
 50 mind during mathematical problem solving. Hess and Polt (1964) were the first to demonstrate that

1 a person's pupils dilate when he or she is solving a difficult mathematical problem. As their multipli-  
 2 cation problems got harder, pupil dilation steadily increased. It was a few years later that saccadic  
 3 movements of the eyes were linked to problem-solving tasks. Yarbus (1967) showed that it is not  
 4 merely visual input that determines eye movement patterns, but task constraints as well. In his exper-  
 5 iment, the same painting (Ilya Repin's *An Unexpected Visitor*) elicited very different eye movement  
 6 scanning patterns (scanpaths) when the viewer was probed with different queries, such as 'What are  
 7 the ages of the people in the painting?', 'What had the family been doing before the visitor arrived?',  
 8 and 'How long had the visitor been away?' (For similar results in individuals with Asperger's  
 9 syndrome, see Benson and Fletcher-Watson, Chapter 39, this volume.)

10 Using the record of saccadic eye movements to uncover the cognitive processes involved in mathe-  
 11 matical problem solving was first systematically explored by Suppes et al. (1982). In testing their proce-  
 12 dural theory of eye movements during addition and subtraction problems, they found that although  
 13 arithmetic problems that require 'carrying' or 'borrowing' are more difficult and take longer than those  
 14 that do not, actually 'carrying' or 'borrowing' values while solving the problem increases eye fixation  
 15 durations only very slightly. When solving algebra problems, it is noteworthy that students do not simply  
 16 fixate each number and symbol left-to-right. Instead, they will sometimes skip values (presumably  
 17 perceiving them parafoveally) and other times they will refixate values (Salvucci and Anderson, 2001).

18 Where we can clearly observe the eyes spending substantially more time fixating difficult parts of  
 19 a math problem is with word problems. Word problems can be written with inconsistent relational  
 20 terms, such as 'Johnny has five cookies, which is three fewer than Mary has. How many cookies does  
 21 Mary have?' This word problem is referred to as 'inconsistent' because the relational term 'fewer  
 22 than' can trick students into using subtraction (instead of addition) to solve the problem. In fact, eye  
 23 movement patterns (such as more regressive saccades back to the relational terms) suggest that this  
 24 is exactly why students are sometimes unsuccessful with inconsistently worded problems (Hegarty  
 25 et al., 1995; Verschaffel et al., 1992).

26 Eye movements during the linguistic delivery of math and logic problems have also been explored  
 27 when there were no visual stimuli at all. For example, Demarais and Cohen (1998) presented partici-  
 28 pants with spoken recordings of transitive inference problems, such as 'a jar of pickles is below a box  
 29 of tea bags; the jar of pickles is above a can of coffee; where's the can of coffee?' While staring at a  
 30 completely blank display, participants tended to make more vertical (than horizontal) eye move-  
 31 ments while hearing those above/below inference problems. When hearing inference problems where  
 32 objects were aligned to the left and right of one another, participants made more horizontal (than  
 33 vertical) eye movements. These results suggest that when people are solving transitive inference  
 34 problems, with no external visual aids, they nonetheless generate some form of spatial mental model  
 35 to assist their logical induction (Byrne and Johnson-Laird, 1989), and they perform eye movements  
 36 consistent with the spatial characteristics of that mental model.

37 Eye movement patterns can also reveal the cognitive processes involved in solving geometry prob-  
 38 lems. Epelboim and Suppes (2001) tracked participants' eye movements while they looked at  
 39 diagrams that had letters indexing the endpoints of lines and centroids of circles, and then some  
 40 angles provided in degrees, and then a particular angle whose size (in degrees) was to be solved. The  
 41 eye movement patterns exhibited a variety of interesting properties. For example, participants tended  
 42 to look away from the diagram when they were performing mental arithmetic (as indicated by their  
 43 verbal protocols). More importantly, the scanpaths revealed a great many redundant sequences  
 44 consisting of many rescans of previously fixated diagram components throughout the solving of the  
 45 problem. This observation is consistent with Epelboim and Suppes's model of geometry reasoning in  
 46 which fixated components of a diagram tend to overwrite existing diagram components in visual  
 47 working memory—suggesting that visual working memory may tend to store a surprisingly small  
 48 amount of information at any one time (see also Ballard et al., 1997).

#### 49 **Mechanical problem solving**

50 Carpenter and Just (1978; see also Just and Carpenter, 1985) were the first to show a relationship  
 51 between eye movements and mental rotation. When participants compare a target shape to a set of

1 alternative rotated shapes, to find the matching one, the eye movement patterns suggest that partici-  
 2 pants go through a few different procedures in sequence. They encode the target shape and *search* for  
 3 a matching component among the shapes, then *transform* or rotate that component in one of the  
 4 shapes, and then *confirm* whether the other components match those in the target shape after this  
 5 mental transformation (Just and Carpenter, 1985). Interestingly, the eye movement data suggested  
 6 that the increased latencies resulting from more complex shape comparisons appeared to have been  
 7 due to longer encoding and confirmation phases, rather than longer transformation or rotation  
 8 phases (Carpenter and Just, 1978).

9 While eyetracking a simplified version of the Tower of London problem, Hodgson et al. (2000)  
 10 also found evidence for a search phase, followed by a transformation phase, and then a confirmation  
 11 phase. Participants started each trial with multiple fixations of the target stack of balls, then multiple  
 12 fixations of the stack of balls that they were to imagine manipulating to match the target, and then  
 13 finishing with fixations of the target stack again. Interestingly, on trials where participants produced  
 14 incorrect answers, their scanpaths were surprisingly conspicuously similar to those of the previous  
 15 trial. Hodgson et al. suggested that participants sometimes get stuck from one trial to the next in an  
 16 oculomotor routine that may not be optimal for the new problem diagram.

17 Mental animation during mechanical reasoning can also be revealed in eye movement data. For  
 18 example, Hegarty (1992; see also Hegarty and Just, 1993) recorded participants' eye movements  
 19 while they looked at diagrams of pulley systems, and found evidence that when people mentally  
 20 animate these diagrams, in order to verify/deny statements about them, their animations are piece-  
 21 meal. Rather than imagining all the pulleys moving at once, as would actually happen when the rope  
 22 threads its way through them, the eye movement patterns suggest that participants mentally animate  
 23 small portions of the pulley system in sequential order, reflecting the causal chain of forces.

24 The same observation was made for eye movement patterns with more complex gear-and-belt  
 25 diagrams. Rozenblit et al. (2002) tracked participants' eye movements while they looked at gear-  
 26 and-belt diagrams in which the leftmost gear was specified to rotate in a particular direction, and it  
 27 connected to several intermediate gears (which alternate the previous direction of rotation) and  
 28 straight belts (which maintain the previous direction of rotation), culminating in a pendulum. The  
 29 participant's task was to guess which direction the pendulum would swing. Practised eye movement  
 30 coders could tell which directions a participant was mentally animating a set of connected gears and  
 31 belts by watching the slow-motion video of the eye movement trace overlaid on the diagram. For  
 32 example, a few fixations near the conjunction of two gears can reveal that the participant was mentally  
 33 animating the left gear clockwise and the right gear counter-clockwise simply by the fact that the  
 34 sequence of fixations went left-to-right on the *upper* portion of the left gear and then left-to-right on  
 35 the *lower* portion of the right gear. The proof for this observation comes especially clear when that  
 36 direction of animation is incorrect and thus allows the coder to accurately predict an incorrect guess  
 37 from the participant on that trial. In fact, Rozenblit et al. showed that even unpractised coders—  
 38 naïve experimental participants watching the slow-motion video of multiple trials (with their final  
 39 portions edited out)—can use the eye movement traces to perform well above chance in predict-  
 40 ing incorrect guesses. Thus, even when it is difficult to identify the specific properties of the eye-  
 41 movement trace that reveal cognition, there is clearly information in those scanpaths that can  
 42 nonetheless be used (even by non-experts) to infer cognitive processes.

### 43 Insight problem solving

44 So far, we have been discussing problem-solving tasks where the formulation of the solution plods  
 45 along relatively linearly. With problems like that, participants tend to have accurate intuitions about  
 46 how close they are to reaching the solution. With insight problems, by contrast, that is not the case  
 47 at all. With insight problems—and there is a wide variety of them—participants usually reach a point  
 48 partway through where they feel like they've run out of ideas, known as an *impasse*. When encour-  
 49 aged to keep trying anyway, some proportion of the participants will eventually achieve *insight*, in  
 50 some cases even widening the eyes and declaring 'Aha!' Notably, on the way toward that 'Aha!'

1 moment, if they are asked to report how close to the solution they think they might be, their ratings  
 2 do not correlate with their actual remaining time-to-solution. Insight problems tend to be difficult  
 3 puzzles, so a substantial proportion of participants never find the solution at all (or they require  
 4 guiding hints from the experimenter to get there). Participants typically describe the solution as  
 5 having come to them ‘suddenly out of nowhere.’ Nonetheless, more subtle measures of performance,  
 6 such as two-alternative forced-choice (Bowers et al., 1990), reaction-time priming (Bowden and  
 7 Jung-Beeman, 1998), and eye-tracking (Knoblich et al., 2001), can often provide inklings of how the  
 8 solution gradually emerges over time. This opacity to subjective reports makes insight problem solv-  
 9 ing particularly intriguing for theories of cognition, and a prime target of eye movement  
 10 methodologies.

11 Before discussing the more standard insight problems, however, it might be useful to examine a kind  
 12 of problem that falls somewhere in between non-insight problems and insight problems. This next  
 13 eyetracking study that opens a window onto a mechanical problem-solving task is not typically consid-  
 14 ered an insight problem, per se, because it does not involve an impasse phase. Participants view a  
 15 sequence of connected gears, are told that the leftmost gear will turn clockwise (or counter-clockwise),  
 16 and then asked which direction the rightmost gear will turn. After several trials of solving these simple  
 17 linear-gear-sequence problems, participants rather suddenly discover a new easy strategy for solving  
 18 them. In fact, sometimes they even shout ‘Aha!’ when they suddenly realize that one need not mentally  
 19 animate each gear to figure out which direction the last one rotates; one can just count whether there  
 20 is an even or odd number of gears (Dixon and Bangert, 2004). If there is an even number of linearly-  
 21 connected gears, then the last gear will rotate in the *opposite* direction of the first gear. If there is an odd  
 22 number of gears, then the last gear will rotate in the *same* direction as the first gear. What is interesting  
 23 about this discovery process is that the statistical structure of the eye-movement patterns presages that  
 24 moment of discovery. That is, over the course of the two to five trials that precede the ‘Aha’ moment  
 25 of figuring out the even-odd strategy, the eye-movement trace shows a statistically reliable increase in  
 26 entropy, a measure of disorder in the data (Stephen et al., 2009). Stephen et al. interpret this as evidence  
 27 that the mental animation strategy is becoming unstable, and is on the brink of yielding to the emerg-  
 28 ing even-odd strategy. On the one trial immediately preceding the trial on which they first use their  
 29 new even-odd strategy, the eye-movement trace shows a statistically reliable *decrease* in entropy. This  
 30 suggests that right before the new strategy is implemented, participants’ scanning behaviour reveals a  
 31 renewed stability and order reflecting the completion of the cognitive restructuring from the old  
 32 mental animation strategy to the newly-adopted even-odd strategy. Thus, eye-movement patterns can  
 33 provide a window into the gradual emergence of a new strategy during problem solving—even predict-  
 34 ing when a new strategy is on its way.

35 This particular view on cognitive processes provided by eye-tracking can even be seen in genuine  
 36 insight problems, which involve both an impasse and then an ‘Aha’ moment (for a more extensive  
 37 review, see Knoblich et al., 2005). Knoblich et al. (2001) tracked participants’ eye movements while  
 38 they attempted to solve/repair Roman numeral arithmetic problems written with matchsticks. For  
 39 example, reposition a single matchstick to make the following equation true:  $IV = III + III$ . (Note  
 40 that all numerals and operators are formed with straight matchsticks.) In this non-insight example,  
 41 one simply needs to change the ‘IV’ to a ‘VI’, and the equation is true. Participants tend to solve that  
 42 one quickly and easily. However, more difficult problems, such as  $III = III + III$ , will often induce an  
 43 impasse; and when the participant discovers the solution, they experience an ‘Aha’ moment. The  
 44 solution for converting that false equation into a true one, involves relaxing an implicitly assumed  
 45 constraint (that only matchsticks forming numerals can be modified).

46 Knoblich et al. (2001) found that, during the impasse phase with the difficult problems, mean fixa-  
 47 tion duration increased significantly. Basically, when participants had run out of ideas, they stared at  
 48 the display without exploring new solution strategies. While that finding shows the impasse stage of  
 49 insight problem solving, this next finding shows how eye movements can reveal the representational  
 50 change that takes place right before achieving insight. As would be expected from an implicit (and  
 51 incorrect) assumption that only matchsticks forming numerals can be modified, participants typically  
 52 began the difficult problems fixating the numerals almost exclusively, rarely fixating the operators.

1 Over the course of attempting to solve these difficult ‘constraint relaxation’ problems, the participants  
 2 who would soon discover the solution (e.g. changing the plus sign to an equal sign with a single move-  
 3 ment of one matchstick) showed a steady gradual increase in proportion of fixations on operators  
 4 prior to their ‘Aha!’ moment.

5 Similar evidence for eye movements revealing the emergent temporal dynamics of a seemingly-  
 6 sudden insight comes from work by Jones (2003). Jones used a diagram-based car park problem,  
 7 where the tricky insight is that, in addition to moving other cars out of the way to form an exit path,  
 8 one’s own car must move to make room for some of those other cars’ movements. Jones operational-  
 9 ized the impasse as a looking time for a particular move that exceeded that participant’s mean look-  
 10 ing time by two standard deviations. He found that all 30 (out of 37) participants who solved the  
 11 problem experienced at least one such impasse. Moreover, the bulk of that impasse was focused on  
 12 the part of the problem that required realizing that one’s own car needed to be moved. Mean looking  
 13 time for that move was five times greater than the mean looking time for other moves. However,  
 14 about one-third of the participants exhibited impasses (long gazes without making any moves) quite  
 15 early on, suggesting that they may have been strategizing multiple moves in advance. In fact, those  
 16 participants with early impasses solved the car park problem faster and with fewer moves than partic-  
 17 ipants who did not exhibit early impasses.

## 18 Eye movements influence problem solving

19 In addition to providing cognitive scientists with a peek into the mental activity involved in problem  
 20 solving, eye movements may themselves be able to instigate insight for a person attempting to solve  
 21 a problem. For example, Grant and Spivey (2003, expt. 1) showed that fixating certain regions of a  
 22 tumour-and-stomach diagram, for Duncker’s classic radiation problem, was correlated with achiev-  
 23 ing insight. The schematic diagram consisted of a solid oval, representing the tumour, with a circum-  
 24 scribing oval representing the stomach lining. While viewing that display, participants were told,  
 25 ‘Given a human being with an inoperable stomach tumour, and lasers which destroy organic tissue  
 26 at sufficient intensity, how can one cure the person with these lasers and, at the same time, avoid  
 27 harming the healthy tissue that surrounds the tumour?’

28 All participants reported experiencing a period of impasse, where they had run out of ideas. Only  
 29 about a third of the participants produced the correct solution (i.e. to use multiple low-intensity  
 30 lasers, which are too weak to damage the skin, and converge their incident rays onto the tumour to  
 31 combine their intensities and burn it away). These successful participants routinely blurted out an  
 32 ‘Aha!’ (or some equivalent) when they discovered this solution. Although the eye movement patterns  
 33 were somewhat similar for successful and unsuccessful participants, one key difference stood out.  
 34 Successful solvers tended to spend more time looking at the stomach-lining portion of the diagram  
 35 than unsuccessful solvers. In fact, the pattern of eye movements that was most closely associated with  
 36 achieving insight with this problem was one in which one or two saccades were made from the exter-  
 37 nal portion of the diagram inward to the tumour (often stopping at the stomach-lining) and then  
 38 back out to another external region. This triangular sequence of saccades, which almost ‘paints a  
 39 sketch’ of multiple incident rays converging on the tumour, was especially common prior to the  
 40 participant achieving insight. Could it be that people who *happened to* produce eye movement  
 41 patterns like that were steered toward the cognitive insight by their sensorimotor patterns priming a  
 42 ‘perceptual simulation’ (Barsalou, 1999) of multiple incident rays? Or is it merely that their cognitive  
 43 processes were gradually forming the correct solution anyway, and this caused their eye movements  
 44 to reflect that nascent solution?

45 In a first step toward addressing this question, Grant and Spivey (2003) conducted a second exper-  
 46 iment in which a computer display animated either the tumour or the stomach-lining, with a one-  
 47 pixel increase in diameter flashing at about 3 Hz. With attention and eye movements drawn toward  
 48 the animated tumour, only a third of those participants solved the radiation problem—the same as  
 49 with the static diagram in their first experiment. However, of the participants whose diagrams had  
 50 the stomach-lining flashing, two-thirds discovered the insight of converging multiple weak lasers

1 onto the tumour. Thus, in this second experiment, attracting participants' attention toward the  
 2 stomach-lining region of the diagram (which is where successful solvers were spontaneously looking  
 3 in the first experiment) *doubled* the solution rate for this difficult insight problem. Note, however,  
 4 that because Grant and Spivey did not track eye movements in this second experiment, it is not clear  
 5 whether it is eye movements, per se, that jump-started this cognitive insight, or whether it might be  
 6 covert spatial attention that did so.

7 Following up on this issue, Thomas and Lleras (2007) used a similar schematic diagram with  
 8 Duncker's radiation problem and added to it a secondary visual-tracking task that, for some subjects,  
 9 *just happened* to make their eyes move in the same triangular fashion that Grant and Spivey (2003)  
 10 first noticed in their eye-movement data. Eye movements were recorded to ensure that participants  
 11 adhered to the visual-tracking task. Importantly, participants whose eye movements were guided (by  
 12 the visual-tracking task) to reproduce this triangular pattern were more than twice as likely to  
 13 discover the solution than participants whose eye movements were driven in neutral or irrelevant  
 14 patterns. Since participants reported not detecting any relationship between the visual-tracking task  
 15 and the radiation problem, Thomas and Lleras (2007) concluded that the eye movements were influ-  
 16 encing the formation of insight in this problem.

17 Interestingly, since covert allocation of spatial attention is often likened to programmed eye move-  
 18 ments that simply aren't executed (e.g. Sheliga et al., 1994), one might expect that patterns of covert  
 19 spatial attention—independent of eye movements—might be equally capable of influencing cogni-  
 20 tion in a diagram-based problem like this. Indeed, Thomas and Lleras (2009) found that when eye  
 21 position was fixed on the tumour, but visual cues directed covert spatial attention to the external  
 22 portion of the diagram and then back to the tumour again and again, the solution rate almost  
 23 tripled—compared to central fixation without attentional cues. Essentially, preparing eye move-  
 24 ments (with or without executing them) that are spatially compatible with the convergence solution  
 25 to the radiation problem is sufficient to prime the perceptual simulation that embodies the cognitive  
 26 insight. Thus, eye movement processes do more than simply *reveal* (to the experimenter) the tempo-  
 27 ral dynamics of cognition during problem solving. Eye movement processes can also *influence* (for  
 28 oneself) the cognitive processes that go into an attempt to solve a problem.

## 29 Conclusion

30 Classical approaches to explaining problem solving, described in the introduction, neglect the  
 31 perceptuomotor processes that are likely involved in problem solving. The eye-tracking work  
 32 described above shows that embodied representations both reflect unfolding problem solving, and  
 33 may directly aid in solving certain kinds of problems. Eye-tracking methodologies are extremely well  
 34 suited to identifying these representations as they occur. For one, the eye-movement record is a  
 35 semi-continuous real-time source of information about cognitive processes, in many ways out of  
 36 conscious strategic control of participants (Liversedge and Findlay, 2000). In addition, as a motor  
 37 system, it maps out spatial coding of the sensorimotor processes themselves (see Spivey et al., 2009,  
 38 for further review).

39 These characteristics guarantee that eye movements will remain, well into the future, an important  
 40 methodological window on high-level cognitive processes, such as problem solving. In conjunction  
 41 with other near-continuous measures of cognition used recently, such as the dynamics of reaching  
 42 and pointing movements (e.g. Spivey and Dale, 2006; Spivey et al., 2005; Stephen et al., 2009), it  
 43 offers investigation into the time-course and fine-grained representational structure of problem  
 44 solving. Spatially and temporally coarser techniques, such as coded video streams or think aloud  
 45 protocols (Ericsson and Simon, 1993) are still applied extensively to complex cognitive processes,  
 46 like knowledge acquisition, problem solving, metacognition, and so on. Advances in the analysis of  
 47 semi-continuous methods continue to offer novel insights that those coarser methods cannot access,  
 48 especially in these high-level contexts (e.g. Graesser et al., 2005).

49 Finally, the theoretical question regarding whether there can be a preferred account of problem  
 50 solving, classical or embodied, is not at present resolvable. In many regards, classically formal

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 Liversedge and  
 Findlay, 2000  
 reference



1 approaches enjoy more currency in this domain (Poole et al., 1997). Recently, some artificial intel-  
 2 lligence research has been integrating symbolic and spatial representations, and solutions to some  
 3 problems come handily with both in operation, compared to either alone (e.g. Wintermute and  
 4 Laird, 2008). This perhaps encourages one to consider that both theoretical schemes may be helpful  
 5 for understanding what guides a problem solver. Such integration will depend on a number of  
 6 important factors, such as the particular problem at hand, problem-solving context, and any poten-  
 7 tial expertise. Eye tracking and other semicontinuous methods will inevitably provide fine-grained  
 8 explorations of these and other factors in the future, contributing to ongoing theoretical debate.

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