

categorical  
Y

# Introduction to Recurrence Quantification Analysis (RQA)

Rick Dale

Department of Communication  
University of California, Los Angeles  
[co-mind.org/atf](http://co-mind.org/atf)

*Nonlinear Methods for Psychological Science*  
APA Advanced Training Institute  
University of Cincinnati



---

---

---

---

---

---

---

---

---

---

[co-mind.org/atf](http://co-mind.org/atf)

additional materials related to dynamical methods for categorical data

---

---

---

---

---

---

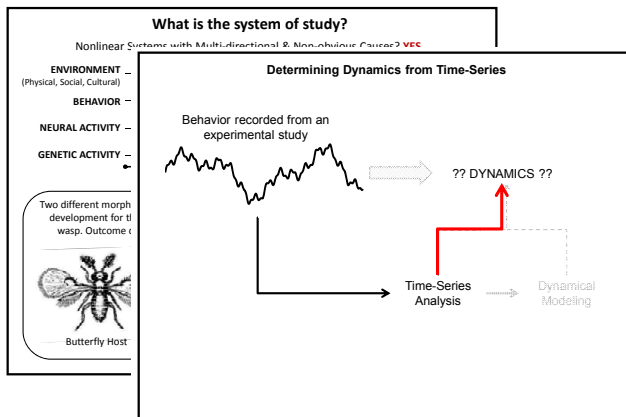
---

---

---

---

from presentation Monday



---

---

---

---

---

---

---

---

---

---

from presentation Monday

### Dynamics & Behavior

- Behavioral dynamics can be measured & quantified
- Behavioral dynamics can be a rich source of information to help understand behavior
- But, we often find ourselves dealing with snapshots of behavior
  - Single measurements, or averages of measures that were collected over time
  - Can be misleading or tell an incomplete story



---

---

---

---

---

---

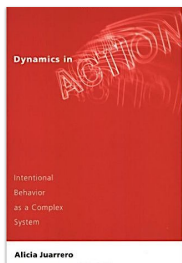
---

---

---

---

Prof. Alicia Juarrero



MIT Press, 1999

...we leverage  
**sequential  
structure of  
behavior**

---

---

---

---

---

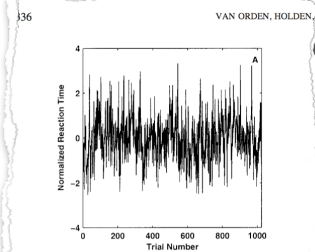
---

---

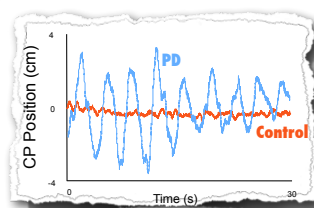
---

---

---



Van Orden, Holden, & Turvey, 2003



Schmit et al. (2006)

---

---

---

---

---

---

---

---

---

---




---

---

---

---

---

---

---

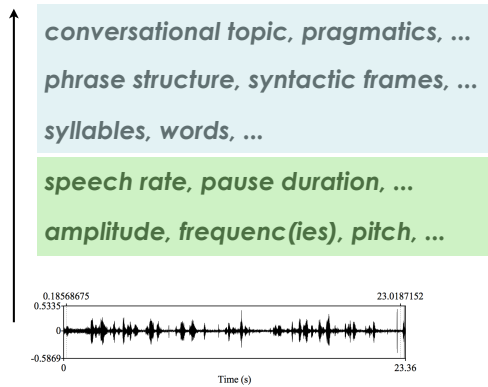
---

---

---

---

## Linguistic Levels...




---

---

---

---

---

---

---

---

---

---

---

## “Higher-Order” States

- Dynamics of the cognitive system produce **higher-order properties** that can be subjected to dynamic methods.
- These measurements are on a **nominal** scale, such as behavioral categories over time (e.g., emotions)
- **Language** is often studied in this manner: sounds, words, sentence structures, topics of conversation, etc.

---

---

---

---

---

---

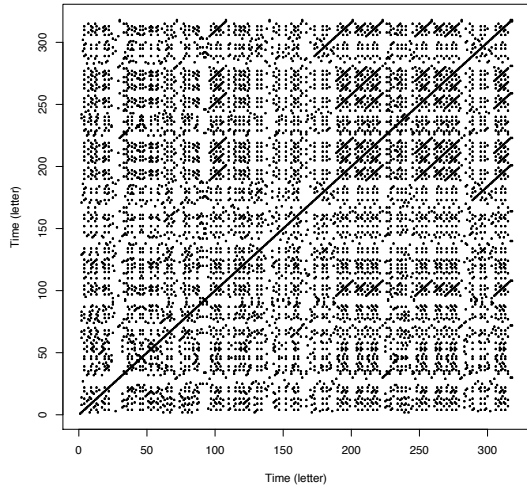
---

---

---

---

---




---

---

---

---

---

---

---

---

---

---

## Outline

1. Time series of higher-order states  
     Analysis of series of behavioral *categories*
2. The recurrence plot (RP) and "textures"
3. Quantifying the plot (RQA)
4. Examples and exercises

---

---

---

---

---

---

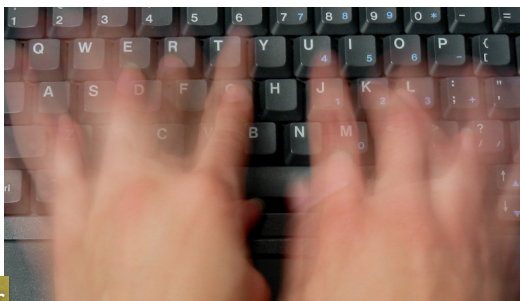
---

---

---

---

### Example...



letter

i am at the word le nonlinear ati this year  
 and enjoy discussing nonlinear and  
 dynamic methods. i also enjoy the practical  
 sessions in which we discuss the research  
 of attendees too.

sentence

---

---

---

---

---

---

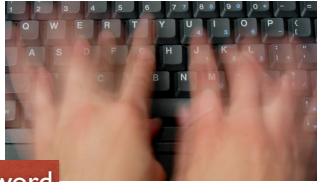
---

---

---

---

## Example...



letter

nonlinear, online, let's hear it for today,  
the content is delicious, fractal in every way.  
nonlinear, nonlinear, rich in data too.

word

let's consult about your research - i dig, do you?

sentence

---

---

---

---

---

---

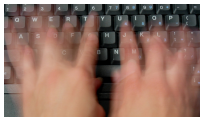
---

---

---

---

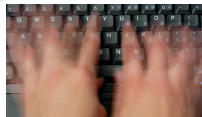
## Example



**Spontaneous**

i am attending the nonlinear at  
this year and i enjoy showcasing  
nonlinear and dynamic methods. i  
also enjoy the practical  
sessions in which we discuss the  
research of attendees too.

spontaneous.txt



**Poetic**

nonlinear, nonlinear, let's hear it for today  
the content is delicious, fractal in every wa  
nonlinear, nonlinear, rich in data too.  
let's consult about your research - i dig, do

poetic.txt

---

---

---

---

---

---

---

---

---

---

## Conversion...

- How do we get **text** into a format that permits use of nonlinear software packages?
- Let's **convert** transcripts to sequence of numeric identifiers:
  - “i am attending the nonlinear ...”  
 $i = 1, \langle \text{space} \rangle = 2, a = 3 \dots$
  - 1 2 3 4 2 3 5 5 ...
- Level of analysis here: **letters**

---

---

---

---

---

---

---

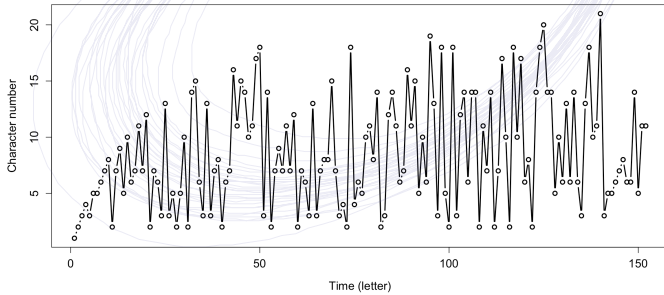
---

---

---

# Trajectories

- Think of the behavior sequences in geometric terms: **trajectories in “category space.”**



---

---

---

---

---

---

---

---

---

---

# Recurrence Plot (RP)

- Bird's-eye view of the system's trajectories through its behavior space.
- With recurrence plots, we visualize various features of this path.
  - How repetitive is a sequence? If repetitive, how long are the repetitions?
  - Lots of short bursts, or fewer but lengthy repeated trajectories?

---

---

---

---

---

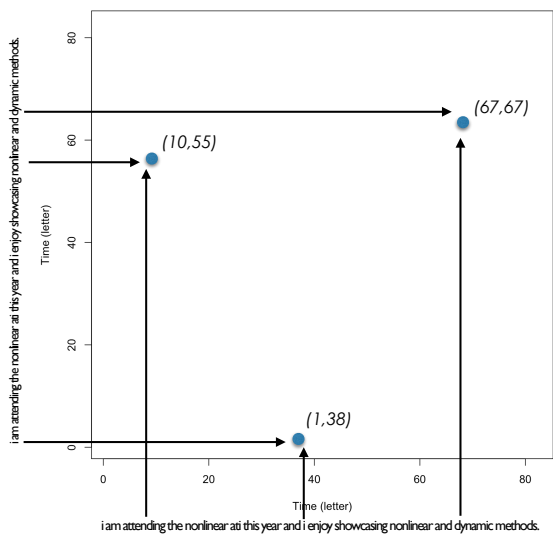
---

---

---

---

---



---

---

---

---

---

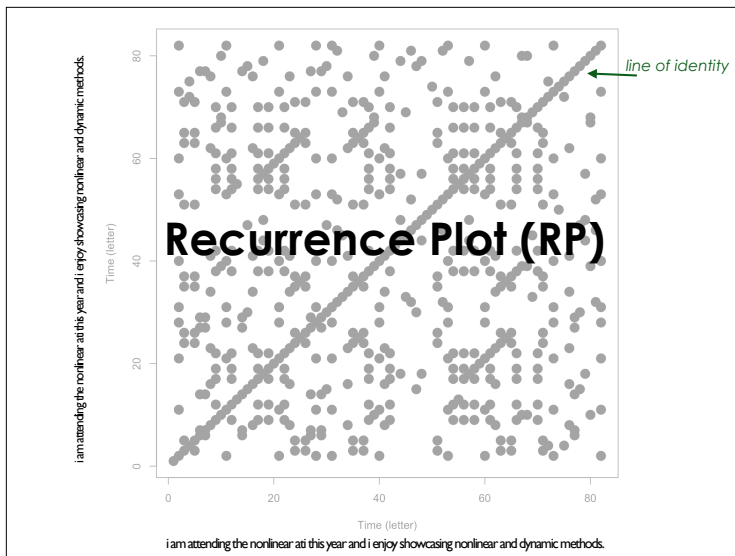
---

---

---

---

---




---

---

---

---

---

---

---

---

## Under the Hood

- If time series is:
 
$$\mathbf{x} = 1, 3, 2, 3, \dots, x_N$$
- RP = set of points  $(i, j)$  such that:
 
$$x_i - x_j = 0$$
- In other words, set of points such that the numeric identifiers have a distance of zero from each other.

---

---

---

---

---

---

---

---

## Textures (Eckmann et al., 1987)

**Spontaneous**

**Poetic**

---

---

---

---

---

---

---

---

## Exercises 1 & 2:

1. Convert Some Text
2. Build Some RPs

---

---

---

---

---

---

---

---

## Quantification

- You can eyeball a plot, but in real contexts we want some **quantification** so that plots (or, e.g., conditions) can be compared.
- Enter: *recurrence quantification analysis* (RQA).
  - These are measures that describe the **extent and distribution** of points on the plot.

---

---

---

---

---

---

---

---

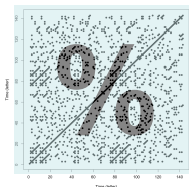
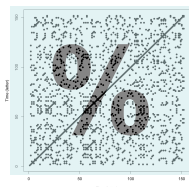
## RQA Measures

### %REC (or, RR)

Spontaneous

Poetic

Recurrence rate (%REC): Total percentage of the plot occupied by points.



7.6%

7.8%

<http://www.recurrence-plot.tk>  $RR = \frac{1}{N^2} \sum_{i,j=1}^N R_{i,j}$

---

---

---

---

---

---

---

---

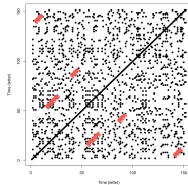


## RQA Measures

### %DET (or, DET)

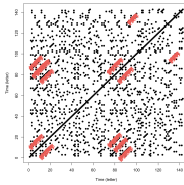
Percent determinism (%DET): Percentage of the points on the plot that fall on diagonal lines (length > 1).

Spontaneous



28.7%

Poetic



31.7%

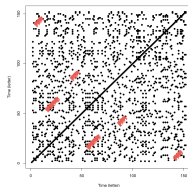
<http://www.recurrence-plot.tk>  $DET = \frac{\sum_{l=1}^N lP(l)}{\sum_{i,j} R_{i,j}}$

## RQA Measures

### MEANLINE

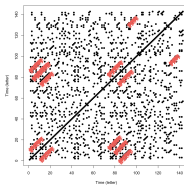
Average diagonal line length (MEANLINE): Average length of diagonal lines on the plot excluding the line of incidence (length > 1).

Spontaneous



3.4

Poetic



4.4

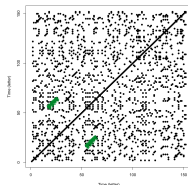
<http://www.recurrence-plot.tk>  $L = \frac{\sum_{l=1}^N l^2 P(l)}{\sum_{l=1}^N l P(l)}$

## RQA Measures

### MAXLINE

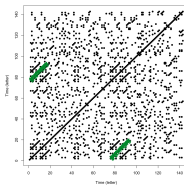
Maximum line length (MAXLINE): The longest diagonal line on the plot (excluding the line of incidence).

Spontaneous



10

Poetic



18

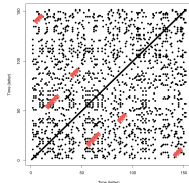
<http://www.recurrence-plot.tk>  $L_{max} = \max\{l_i; i = 1 \dots N_l\}$

## RQA Measures

### ENTROPY

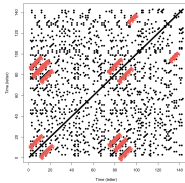
Entropy (*ENTROPY*): The entropy of the distribution of diagonal lines on the plot (how much "disorder" is there in the sequences).

Spontaneous



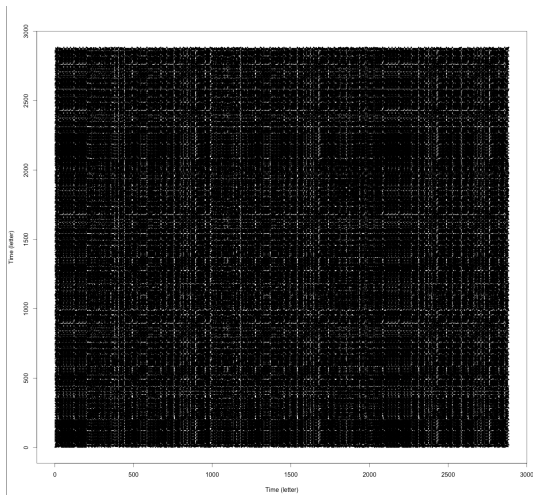
0.7

Poetic



1.1

<http://www.recurrence-plot.tk>  $ENTR = - \sum_{l=1}^N p(l) \ln p(l)$



## Three Important Concepts

- Parameters important for later...
  - embedding dimension,
  - delay,
  - radius
- These parameters will be *crucial* when we apply these RQA methods to *continuous* data (e.g., posture).

# Embedding Dimension

- Interpretation in categorical data: how many states must match in order to count it as a recurrence.
- In previous analysis, dimension = 1
- What about 2? 3?
- *Window, vector, sequence, etc.*

---

---

---

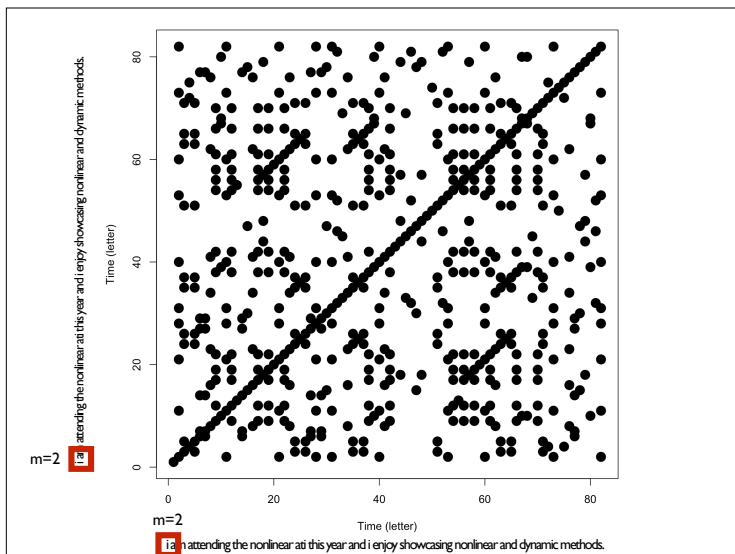
---

---

---

---

---



---

---

---

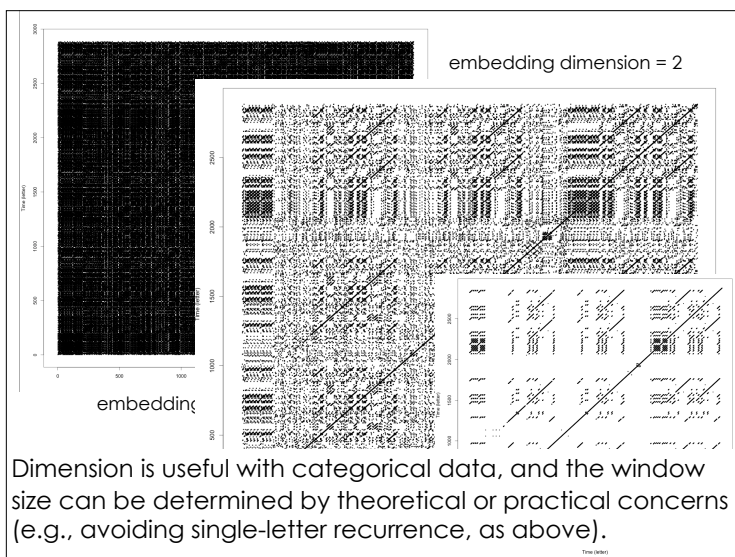
---

---

---

---

---



Dimension is useful with categorical data, and the window size can be determined by theoretical or practical concerns (e.g., avoiding single-letter recurrence, as above).

---

---

---

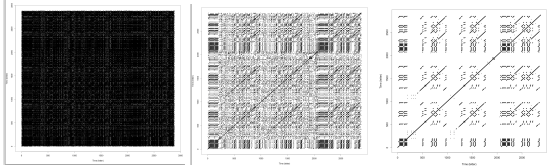
---

---

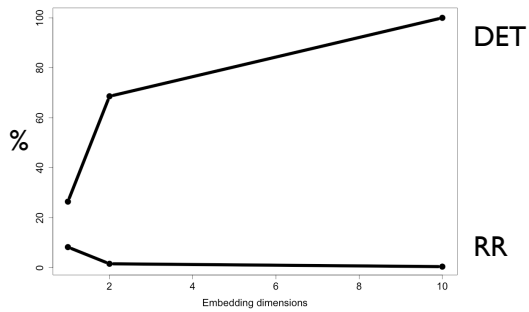
---

---

---



1      2      ...      10




---

---

---

---

---

---

---

---

---

---

---

---

## Delay

- With most categorical data (behavior sequences, linguistic sequences, etc.) temporal ordering should probably be preserved (delay = 1).
- NB: Situation more complex with continuous data (tomorrow).

---

---

---

---

---

---

---

---

---

---

---

---

## Radius

- The distance between units (or windows if dimension > 1) required in order to count  $(i, j)$  as a *recurrent point*.
- With category data: radius = 0.
- NB: Situation more complex with continuous data.

---

---

---

---

---

---

---

---

---

---

---

---

## Radius



---

---

---

---

---

---

---

---

---

---

## But, with categories...

- With most data of nominal codes (e.g., letters, words, etc.), the following parameters often suffice:
  - Embedding dimension: 1
  - Delay: 1
  - Radius: .0001

---

---

---

---

---

---

---

---

---

---

## Exercise 3: Quantify Some Plots

---

---

---

---

---

---

---

---

---

---

## Outline

- Time series of higher-order states
  - Analysis of series of behavioral *categories*
- The recurrence plot (RP) and “textures”
- Quantifying the plot (RQA)
- Examples and exercises

---

---

---

---

---

---

---

---

---

---

## Outline

- Time series of higher-order states
  - Analysis of series of behavioral *categories*
- The recurrence plot (RP) and “textures”
- Quantifying the plot (RQA)
- Examples and exercises
- **Extensions and applications**

---

---

---

---

---

---

---

---

---

---

## Extensions and Applications

- Windowed recurrence analysis
- Fresh work: dynamics of texts
- RQA as “dynamical NLP”

---

---

---

---

---

---

---

---

---

---

# Extensions and Applications

- **Windowed recurrence analysis**
- Fresh work: dynamics of texts
- RQA as “dynamical NLP”

---

---

---

---

---

---

---

---

Nonuniformity in behavioral dynamics

Behavioral “modes”  
Stable, but temporary, functional structures



---

---

---

---

---

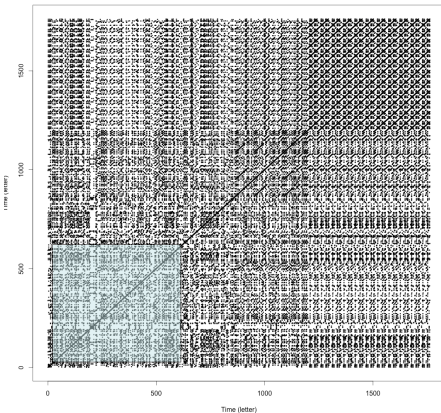
---

---

---

## Windowed Recurrence

embedding dimension = 3



Obtain several new time series of recurrence measures as they change across windows

---

---

---

---

---

---

---

---

## Coco & Dale, 2014

- With Dr. Moreno Coco  
University of Edinburgh
- **R** package for categorical recurrence (adaptable for continuous recurrence)
  - Basic (C)RQA measures
  - Diagonalwise recurrence (tomorrow afternoon)
  - Windowed recurrence measures

---

---

---

---

---

---

---

---

## Cattail Down

by MeWithoutYou

headed east out of st. paul,  
we stopped for water.  
rested in the cemetery,  
watched the mississippi.  
running out of food stamps,  
found a bag along the footpath  
off highway 61 filled with  
what looked like marijuana.  
(don't worry mom, we left it there)  
hopped a grainrail out of pig's eye  
toward milwaukee, a deer  
between the tower and the tracks,  
saw right through us.  
said, "you don't know where you came  
you don't know where you're going,  
you think you're you-

---

---

---

---

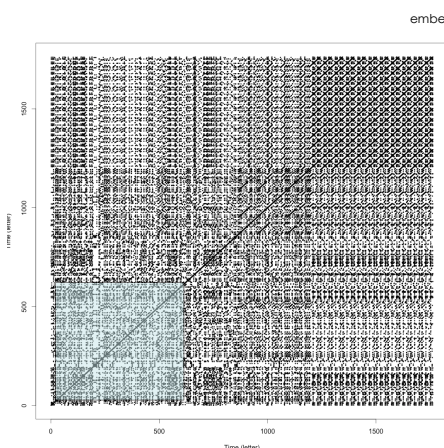
---

---

---

---

## Windowed Recurrence



Obtain several  
new time series  
of recurrence  
measures as  
they change  
across windows

---

---

---

---

---

---

---

---



# runcrqa in R

```
runcrqa( ts1,  
         ts2,  
         par )  
  
# par = list of parameters  
par = list( type = 2, step = 1,  
           windowsize = 50,  
           lagwidth = 40,  
           method = "window",  
           datatype = "categorical",  
           thrshd = 8 )
```

---

---

---

---

---

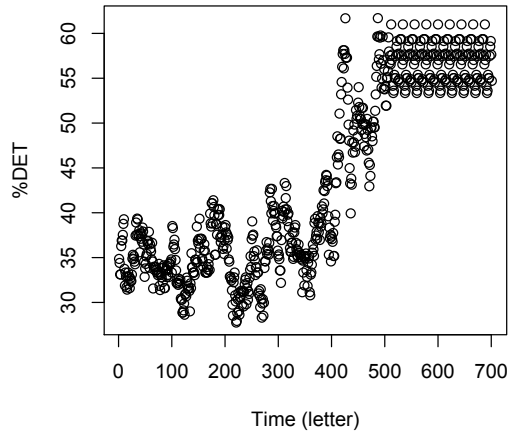
---

---

---

---

---



---

---

---

---

---

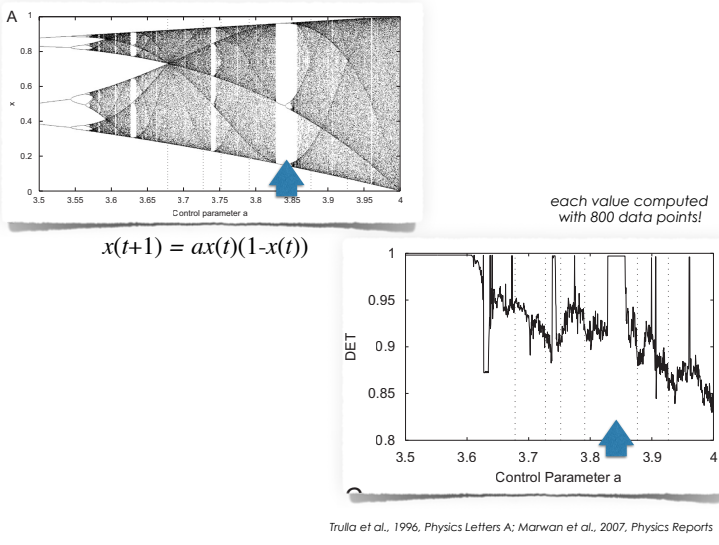
---

---

---

---

---



---

---

---

---

---

---

---

---

---

---

## Extensions and Applications

- **Windowed recurrence analysis**
- Fresh work: dynamics of texts
- RQA as “dynamical NLP”

---

---

---

---

---

---

---

---

---

---

## Extensions and Applications

- Windowed recurrence analysis
- **Fresh work: dynamics of texts**
- RQA as “dynamical NLP”

---

---

---

---

---

---

---

---

---

---

## RQA for Text Analysis

- **Genre** identification in educational data mining contexts.
- Do “**textual dynamics**” differ across history, science, etc. texts?
- Do these dynamic patterns correlate with accessibility, learning gains, etc.?

---

---

---

---

---

---

---

---

---

---



# Extensions and Applications

- Windowed recurrence analysis
- Fresh work: dynamics of texts
- **RQA as “dynamical NLP”**

---

---

---

---

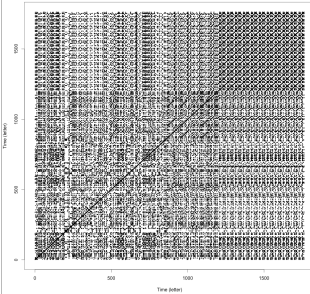
---

---

---

---

## RQA as “Dynamical NLP”



- Can be used inside an interpretive framework anchored to “**complex dynamical systems.**”
- When interpreted simply, can be used as a **technique** for obtaining **descriptive quantities** for behavioral streams

---

---

---

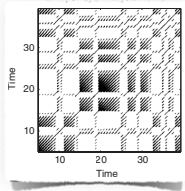
---

---

---

---

---



RQA	NLP Connection
<i>DET</i>	compressibility ratio
<i>RR</i>	co-occurrence or frequency
<i>MAXLINE</i>	longest common subsequence (LCS)
<i>ENTROPY</i>	n-gram variability

---

---

---

---

---

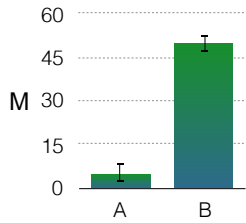
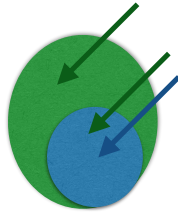
---

---

---

# Nonlinear?

nonlinear **system**  
nonlinear **statistics**  
linear **statistics**



**RQA as “generalized  
[auto-/]cross-  
correlation...”**

Recurrence plots for the analysis of complex systems

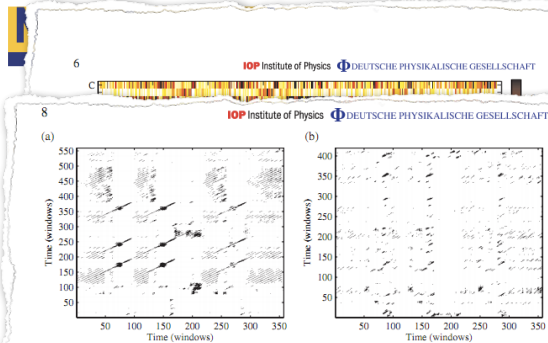
Norbert Marwan\*, M. Carmen Romano, Marco Thiel, Jürgen Kurths

Nonlinear Dynamics Group, Institute of Physics, University of Potsdam, Potsdam 14415, Germany

Accepted 3 November 2006  
Available online 12 January 2007  
editor: I. Procaccia

# Discursis (Angus et al.)

Created at the University of Queensland, Australia, Discursis is a computer-based tool for analysing human communication. Discursis can assist practitioners in understanding the structure, information content, and inter-speaker relationships that are present within input data.



**Figure 3.** CRPs for the song *Day Tripper* as performed by The Beatles, taken as song X, versus two different songs, taken as song Y. These are a cover made by the group Ocean Colour Scene (a) and the song *I've Got a Crush on You* as performed by Frank Sinatra (b). Parameters are  $m = 9$ ,  $\tau = 1$ , and  $\kappa = 0.08$ .

**Exercise 4 (wrapping up):**  
**Extended Exercises on RQA**

---

---

---

---

---

---

---

---

---

---

---